Titles and Abstracts

Shimura varieties, torsion classes, and Galois representations

Keynote talk
Thursday
2pm
LT3

Dr. Ana Caraiani

Imperial College London

In this talk, I will describe joint work in progress with Peter Scholze on torsion in the cohomology of certain unitary Shimura varieties. I will explain how the theory of perfectoid spaces and p-adic Hodge theory give new insights into the geometry and cohomology of Shimura varieties. I will also mention applications of these results to elliptic curves over imaginary quadratic fields, joint with Allen, Calegari, Gee, Helm, Le Hung, Newton, Scholze, Taylor, and Thorne.

Galois action on elliptic curves

Research talk
Friday
10:30am
LT5

Nirvana Coppola

University of Bristol

The starting point of my work is the study of elliptic curves defined over a p-adic field and ℓ-adic Galois representations attached to them. In particular an interesting problem is to describe the action and the possible image of the inertia group under these representations. This action depends first of all on the reduction type of the curve, for example it is trivial if and only if the curve has good reduction modulo p. Another key factor is the residue characteristic p of the field: if p is different from 2 and 3, it is relatively easy to describe (it is cyclic). If p is either 2 or 3, it is convenient to express the image of inertia in terms of the Galois group of a certain finite extension, giving explicitly the generators for it.

On the theory of higher rank Euler systems

Overview talk
Friday
12pm
LT5

Alexandre Daoud

King's College London

I will report on the recent work of David Burns and Takamichi Sano regarding the construction of higher rank Euler systems for p-adic representations. In particular, if we fix a number field K, a p-adic representation T of K and an abelian pro-p extension $\mathcal{K}/K$ (such that the data $(K, T, \mathcal{K})$ satisfies some not-to-strong hypotheses), their methods lead to the construction of a canonical (and typically large) $\mathbb{Z}_p[[\mathrm{Gal}(\mathcal{K}/K)]]$-module of higher rank Euler systems. If time permits, I will outline how in the classical setting of the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$, the known validity of the eTNC implies that this construction recovers the cyclotomic units. The main reference for this talk is the paper titled "On the theory of higher rank Euler systems, Kolyvagin systems and Stark systems" by Burns–Sano.

Tame Galois module structure revisited

Research talk
Thursday
10:30am
J11

Fabio Ferri

University of Exeter

This is joint work with Cornelius Greither. A number field K is Hilbert–Speiser if all of its tame abelian extensions L/K admit NIB (normal integral basis). It is known that $\mathbb{Q}$ is the only such field, but when we restrict $\mathrm{Gal}(L/K)$ to be a given group G, the classification of G-Hilbert–Speiser fields is far from complete. In this talk, we present new results on so-called G-Leopoldt fields. In their definition, NIB is replaced by "weak NIB" (defined below). Most of our results are negative, in the sense that they strongly limit the class of G-Leopoldt fields for some particular groups G, sometimes even leading to an exhaustive list of such fields or at least to a finiteness result. In particular we are able to correct a small oversight in a recent article by Ichimura concerning Hilbert–Speiser fields.

Hilbert modular Eisenstein congruences

Research talk
Thursday
5:10pm
J11

Dan Fretwell

University of Bristol

(Joint work with Cathy Hsu and David Spencer)

It is often the case that there is a congruence mod p between the Fourier coefficients of a cusp form (mysterious thingamabob) and an Eisenstein series (well understood thingamabob). The first such congruence can be found in the work of Ramanujan, who remarkably observed that $\tau(n) \equiv \sigma_{11}(n) \pmod{691}$. Here $\tau(n)$ is the coefficient of q^n in the power series expansion of $q\prod(1-q^n)^{24}$ and $\sigma_{11}(n)$ is the 11th power divisor sum of n. Sounds bizarre... right?

Over the last 50 or so years Ramanujan's phenomenon has been observed in much higher generality, and so the theory of Eisenstein congruences has been born. This has blossomed into a collection of far reaching results/conjectures about modular and automorphic forms modulo p, which in turn have lead to various applications in and including Iwasawa theory, the theory of Galois representations and the Bloch–Kato conjecture.

In this talk I will give a brief overview of various classical Eisenstein congruences and discuss work in progress on the generalization of these results to the Hilbert modular case (i.e. modular forms over a totally real field).

Abelian varieties and their endomorphism algebras

Overview talk
Thursday
4:40pm
LT3

Pip Goodman

University of Bristol

We'll begin with an overview of abelian varieties and their endomorphism algebras. Afterwards, we'll see how the action of Galois imposes restrictions on the endomorphism algebra of a given abelian variety.

Modular forms near the boundary of weight space

Research talk
Friday
4:30pm
LT5

Daniel Gulotta

University of Oxford

Let p be a prime, and let N be a positive integer not divisible by p. Some nice patterns appear in the U_p-eigenvalues of the spaces of modular forms $S_k(\Gamma_1(Np^r))$ for $r \geq 2$. Coleman pointed out that these patterns could be partially explained by the existence of a space of "p-adic modular forms with $\mathbb{F}_p((T))$ coefficients". I will construct such a space. In the process, I will give an answer to the question "What does it mean for a function $\mathbb{Z}_p \to \mathbb{F}_p((T))$ to be locally analytic?

Brauer groups of surfaces

Research talk
Friday
3:30pm
F20

Damián Gvirtz

Imperial College London

The Brauer group of an algebraic variety is an important invariant in the obstruction of rational points. To understand it, one can find a filtration into several parts, constant, algebraic and transcendental. The third one is the least understood. Relying on homological algebra work by J.-L. Colliot-Thélène and A. Skorobogatov and complex multiplication on K3 surfaces by Rizov-Valloni, I will show how to fully classify the transcendental part of diagonal quartic surfaces over $\mathbb{Q}$ and sketch further cases. The case of diagonal quartics is joint work with A. Skorobogatov.

Modular points on elliptic curves

Research talk
Thursday
11:30am
J11

Richard Hatton

University of Nottingham

In the arithmetic of elliptic curves, we are interested in the construction of points on an elliptic curve. In particular, it has been shown that we are able to bound certain Selmer groups using modular points, specifically the use of Heegner points by Kolyvagin and Self points by Wuthrich. We will define these points and will show how they can be used to create the bounds and its generalisations.

Perfectoid modular forms

Overview talk
Friday
3:30pm
LT5

Ben Heuer

King's College London

If one compares q-expansions of different modular forms defined over the integers for varying weight, one may find that there are curious mod p congruences occuring in the Fourier coefficients. In the 1970s, Serre studied such congruences by considering p-adic limits of q-expansions of modular forms – the theory of p-adic modular forms was born. Shortly after, Katz reinterpreted this definition in terms of p-adic moduli spaces of elliptic curves.

In recent years, perfectoid spaces have been used to offer a new perspective on these p-adic moduli spaces, and on p-adic modular forms in particular. Moreover, this language allows one to define new spaces of "perfectoid" modular forms. In this talk, we will discuss some aspects of classical modular forms that can be studied using this perfectoid perspective.

Level lowering for GL_n via p-adic uniformisation

Research talk
Friday
11:30am
LT5

Pol van Hoften

King's College London

Mazur's principle states that a cusp form of level $\Gamma_0(pN)$ is congruent (mod ℓ) to a cusp form of level $\Gamma_0(N)$ if and only if the mod ℓ representation is unramified at p. This was generalized to Hilbert modular forms by Carayol, Jarvis and Fujiwara, the latter using p-adic uniformisation of Shimura curves. I will discuss my attempts to generalize this to automorphic forms for GL_n using p-adic uniformisation of (unitary) Shimura varieties.

Higher Eisenstein congruences

Research talk
Thursday
3:20pm
J11

Catherine Hsu

University of Bristol

In his seminal work on modular curves and the Eisenstein ideal, Mazur studied the existence of congruences between certain Eisenstein series and newforms, proving that Eisenstein ideals associated to weight 2 cusp forms of prime level are locally principal. In this talk, we re-examine Eisenstein congruences, incorporating a notion of "depth of congruence," in order to understand the local structure of Eisenstein ideals associated to weight 2 cusp forms of squarefree level.

Irreducible components of local deformation spaces

Research talk
Thursday
12:00pm
F20

Ashwin Iyengar

King's College London

In a paper by Colmez et. al, the authors explicitly describe the irreducible components of the generic fiber of a local Galois deformation ring in the $\ell = p$ case. In particular, they focus on deformations of the trivial representation $\overline{\rho} : G_{\mathbb{Q}_2} \to \mathrm{GL}_2(L)$ for L a finite extension of $\mathbb{Q}_2$. I will describe my current attempts to extend their methods to an arbitrary finite extension $K/\mathbb{Q}_p$ for any p, and to GL_n for arbitrary n.

Principal bundles in Diophantine geometry

Keynote talk
Friday
2pm
LT5

Prof. Minhyong Kim

University of Oxford

Complex equiangular lines and the Stark conjectures

Research talk
Thursday
12:00pm
J11

Gene Kopp

University of Bristol

We describe an application of algebraic and analytic number theory to a long-standing open problem in design theory and quantum mechanics. The existence of a set of d^2 pairwise equiangular complex lines (a SIC-POVM) in d-dimensional Hilbert space is currently known only for a finite set of dimensions d. We prove that, if there exists a set of real units in a certain ray class field (depending on d) satisfying certain congruence conditions and algebraic properties, a SIC-POVM may be constructed when d is an odd prime congruent to 2 modulo 3. We give an explicit analytic formula that we expect to yield such a set of units. Our construction uses values of derivatives of zeta functions at $s = 0$ and is closely connected to the Stark conjectures over real quadratic fields. We will work through the example $d = 5$ in detail to help illustrate our results and conjectures.

Arithmetic invariant theory

Jef Laga

University of Cambridge

Overview talk
Thursday
3:20pm
LT3

Let G be a reductive group over a field k and let V be a linear representation of G. Then the k-algebra $k[V]^G$ of G-invariant polynomials on V is of finite type and we can define the quotient $V//G = \text{Spec } k[V]^G$ together with a quotient map $\pi : V \to V//G$. This method of taking quotients of affine varieties falls under geometric invariant theory. If k is not separably closed, the study of $G(k)$-orbits in the fibers of π over k-points of $V//G$ provides an additional complexity, and often is closely related to arithmetic problems. In this talk I will try to give some examples of how this approach leads to interesting results on the arithmetic statistics of algebraic curves.

L-functions of CM elliptic curves

Marius Leonhardt

University of Cambridge

Overview talk
Friday
10:00am
LT5

An elliptic curve has complex multiplication (CM) if its endomorphism ring is bigger than the integers. This innocent looking definition has many fascinating number-theoretic applications. Amongst them is writing the L-function of the elliptic curve as a product of Hecke characters.

This talk is meant to be an introduction to the theory of complex multiplication. We start by explaining basic properties of CM elliptic curves. Then we construct the associated Hecke characters using (a version of) the so-called main theorem of complex multiplication. Finally, we define the involved L-functions and show their equality.

Modular generating series for Picard modular surfaces

Research talk
Friday
10:30am
LT6

Rob Little

University of Durham

Geometric maps between spaces of modular forms have been studied since the foundational work of Shintani and Shimura on theta lifts between spaces of elliptic modular forms. Throughout the 1970s and 80s, this was regularised into a very general theory by (among others) Don Zagier, Stephen Kudla, John Millson, Friedrich Hirzebruch, James Cogdell, into a theory of theta lifts between modular forms for any two compatible linear groups; we look in particular at the case of the split unitary group $\mathrm{SU}(2,1)$. In this case the lift is taking place on a Picard modular surface, and we look at ways of extending the lift from compactly supported forms to generic differential forms on the surface; this work uses topology, geometry, arithmetic and classical modular forms.

Anabelian geometry

Overview talk
Friday
12:00pm
LT6

Martin Lüdtke

Goethe University Frankfurt

I'll give a brief overview of the field of anabelian geometry, starting with the Neukirch–Uchida theorem and presenting Grothendieck's anabelian conjectures from his letter to Faltings and some more recent results.

An overview of the Hasse norm principle and some recent results

Overview/ research talk
Friday
4:00pm
F20

André Macedo

University of Reading

Given an extension K/k of global fields, we say that the Hasse norm principle holds if every element of k^* which is a norm everywhere locally is in fact a global norm from K^*. In this talk, I'll present Tate's cohomological description of the knot group (an object measuring the failure of the Hasse norm principle) for Galois extensions as well as the main tools to determine this group in the non-Galois case. I'll describe the toric interpretation of this arithmetic principle and how it relates to the weak approximation property on the associated norm one tori. I'll also talk about some recent work of mine in providing an algebraic characterization of this principle in several non-abelian families.

Garrett's conjecture for metaplectic Eisenstein series

Research talk Thursday 4:40pm J11

Salvatore Mercuri

University of Durham

The decomposition of the space of garden-variety modular forms into a direct sum of the cusp forms and the Eisenstein series is well known. What's not so trivial is whether this decomposition also respects algebraicity of the Fourier coefficients of the modular form. The aim is to find a particular algebraic extension of the rationals L whereby if f has coefficients in L, then it can be written as a sum of a cusp form and an Eisenstein series, each having coefficients in L. This was proven in the case of full level Siegel modular forms of integral weight by Harris (1981) using a pretty nifty method which amounts to proving a case of the pretty general Garrett conjecture (1984). This case states that if a cusp form f has algebraic coefficients then the so-called Klingen Eisenstein series associated to f does as well. In this talk, we'll adapt Harris' method to prove this conjecture for Siegel modular forms of half-integral weight of arbitrary level and show how this allows us to determine an algebraic decomposition of the space of such modular forms.

Parity of ranks of abelian varieties

Research talk Thursday 10:30am F20

Adam Morgan

University of Glasgow

For an abelian variety A over a number field K, a consequence of the Birch and Swinnerton-Dyer conjecture is the parity conjecture: the global root number of A determines the parity of its rank. We will survey what is known about the parity conjecture and explain how one may prove (a variant) of this conjecture over quadratic extensions of the original number field.

Brauer–Manin obstruction for rational surfaces

Research talk Friday 4:30pm F20

Masahiro Nakahara

University of Manchester

A variety over a number field satisfies the Hasse principle if it satisfies the local-to-global principle for rational points. This principle can already fail for rational surfaces. It is conjectured that all failures of Hasse principle for rational surfaces are given by the Brauer–Manin obstruction. We study this obstruction for the case of del Pezzo surfaces of degree 2.

The eigencurve at weight one points and units in number fields

Research talk
Friday
4:00pm
LT5

Alice Pozzi

University College London

In 1973, Serre observed that the Hecke eigenvalues of Eisenstein series can be p-adically interpolated. In other words, Eisenstein series can be viewed as specializations of a p-adic family parametrized by the weight. In 1998, Coleman and Mazur defined the eigencurve, a rigid analytic space classifying much more general p-adic families of Hecke eigenforms. The local nature of the eigencurve is well understood at points corresponding to classical cusp forms of weight $k \geq 2$, while the weight one case is far more intricate. In this talk, we discuss the geometry of the eigencurve at weight one points. In particular, we explain how the failure of etaleness at weight one points can be exploited to construct units in certain number fields.

Kurokawa–Mizumoto congruences and degree 8 L-functions

Research talk
Thursday
3:50pm
J11

Angelo Rendina

University of Sheffield

The "algebraic part" of rightmost critical L-value associated to the Delta cusp form has a factor of the large prime 691 in its denominator. This prime is also a factor of the 12th Bernoulli number, and is the modulo of congruence between the Hecke eigenvalues of the weight 12 Eisenstein series and the Delta cusp form itself: this is in accordance with a special case of the Bloch–Kato conjecture. In a joint paper with N. Dummigan and B. Heim, we show that the same phenomenon occurs when considering the spinor L-function associated to the tensor product of an elliptic cusp form and a Siegel cusp form, where the latter is congruent to a Saito–Kurokawa lift modulo a large prime dividing the algebraic part of the L-function associated to the corresponding pre-lift.

Arithmetic statistics through graded Lie algebras

Research talk
Thursday
3:50pm
LT3

Beth Romano

University of Cambridge

I will talk about recent work with Jack Thorne, in which we analyze a grading on a Lie algebra of type E_8 to find the average size of the 3-Selmer group for a family of genus-2 curves.

Multiple zeta values and modular forms

Overview talk
Thursday
10:00am
J11

Alex Saad

University of Oxford

Multiple zeta values (MZVs) are natural generalisations of the values of the Riemann zeta function at integer arguments, and satisfy a wealth of relations and identities. Some of these, due to Ramanujan, provide a "modular" expression for odd zeta values in terms of Eichler integrals of Eisenstein series. We will explore why such formulae exist, and if we can expect similar generalisations for all MZVs, by interpreting these objects within the theory of motivic Galois groups. This is ongoing work relating to the speaker's PhD, supervised by Francis Brown.

The Lang-Trotter conjecture and generalizations

Overview/ research talk
Thursday
5:10pm
LT3

Vlad Serban

University of Vienna

I will give a brief exposition of the Lang–Trotter heuristic which concerns the number of primes p with a fixed trace of Frobenius at p; here the action is coming from the torsion of an elliptic curve E. Then I will explain what happens when one generalizes this prediction to products of elliptic curves.

Equivariant Birch and Swinnerton-Dyer conjecture

Overview talk
Thursday
10:00am
F20

Daria Shchedrina

University of Nottingham

I will tell about possible ways to generalize the conjecture of Birch and Swinnerton-Dyer. We will start with definition of L-function of elliptic curve and then come to formulation of the BSD conjecture, by the end of talk I will tell about some techniques helpful to approach one of parts of the BSD and in case if there is be time left a possible generalization.

On Fermat equation over some quadratic imaginary fields

Research talk
Thursday
11:30am
F20

George Turcas

University of Warwick

Assuming a deep but standard conjecture in the Langlands programme, we prove Fermat's Last Theorem over $\mathbb{Q}(i)$. Under the same assumption, we will discuss Fermat's equation $a^p + b^p + c^p = 0$, for prime exponents p, when a, b, c belong to any of the nine quadratic imaginary fields of class number one. Part of the results can be found at https://doi.org/10.1007/s40993-018-0117-y.

Gauss' conjecture – peculiar proof and progress

Overview talk
Friday
11:30am
LT6

Sadiah Zahoor

University of Sheffield

One of the classical problems in number theory is to determine, for a given positive integer, all possible fundamental discriminants D of imaginary quadratic fields with associated class number $h(D)$ equal to the given positive integer. Gauss had conjectured in 1801 that the class number tends to infinity as the fundamental discriminant tends to minus infinity for imaginary quadratic fields. The proof of this conjecture involved one of the most peculiar usages of the generalised Riemann hypothesis. The talk shall cover from the highlights of the peculiar proof and development about the conjecture — both ineffective and effective versions to the recent progress by M.Watkins.

Shintani lifting

Research talk
Friday
10:00am
LT6

Di Zhang

University of Sheffield

In this talk I will recall Shintani's method to construct modular forms of half-integral weight out of classical modular forms via the theta correspondence. Then I will explain how to generalize this to lift Bianchi modular forms to Siegel modular forms and discuss what I can prove about relating the Fourier coefficients of this lift to special L-values of the Bianchi modular forms.
